WHAT CAN BE CONCLUDED ABOUT SEISMIC HISTORY FROM BROKEN AND UNBROKEN SPELEOTHEMS?

C. LACA VE and M. G. KOLLER
Résonance Ingénieurs-Conseils SA, 21 rue Jacques Grosselin,
CH-1227 Carouge (Genève), Switzerland
corinne.lacave@resonance.ch

J. J. EGOZCUE
Universitat Politècnica de Catalunya, Barcelona, Spain

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Is it really plausible that earthquakes break speleothems? May unbroken speleothems prove that no strong earthquake has ever occurred during a certain period of time? The mechanical behaviour of speleothems has been investigated through static bending tests performed on stalactites and soda straws. These tests give an indication not only of the mean tensile resistance, but also — more importantly — of its variation. In fact, it is this variation that makes it difficult to estimate the acceleration necessary to break an individual speleothem. That is why a statistical approach is mandatory. The potentially most vulnerable unbroken as well as broken stalactites were measured in a pilot cave (Milandre, Switzerland). Four classes of stalactites were defined, according to their shapes. For each of these classes, a vulnerability curve (probability of breaking as a function of peak ground acceleration) was obtained by means of a Monte Carlo simulation. Dynamic amplification as well as heterogeneity of bending resistance within each speleothem were taken into account. Finally, an original statistical approach, valid for incomplete and imprecise data, was developed. This approach allowed to estimate the probability that at least one moderate earthquake has occurred in the past.

Keywords: Long return periods; paleoseismicity; speleothems; stalactites; vulnerability.

1. Introduction

Many reports that interpret broken speleothems (soda straws, stalactites and stalagmites) as indicators of past earthquakes can be found in various literature. A concise overview of what has been published so far can be found in Forti [1997; 1998]. Most of these publications are purely descriptive from a mechanical point of view. However, several efforts have been undertaken to date broken speleothems by means of radiometric methods [Postpischl et al., 1991]. Before broken speleothems can be related to earthquakes, it is obvious that all other possible breakdown causes must be discounted. And this is not an easy task [Delange & Guendon, 1998, Gilli, 1999]. Gilli [1999] further cites high seismicity areas in Costa Rica and Japan where
no evidence of broken speleothems have been seen. However, these caves may actually contain no vulnerable speleothems.

The key question is whether it is really plausible that earthquakes break speleothems. The complementary question is whether unbroken speleothems may prove that no strong earthquake has ever occurred throughout their lifetime. To the knowledge of the present authors, the publication of Gilli et al. [1999] presents the first attempt of quantitatively looking at the mechanical behaviour of speleothems during an earthquake. Cadorin et al. [2001] performed static and dynamical bending tests on four broken stalagmites of the Hotton cave in Belgium in order to determine the calcite rupture stress. They found a minimum tensile rupture stress of 0.42 MPa. Based on this value, they calculated the acceleration that would have been necessary to break 34 broken stalagmites observed in the Hotton cave. A 1 m high stalagmite, 50 mm in diameter, would be expected to break at 2 m/s$^2$. For all others, the acceleration values varied between 9 m/s$^2$ and 667 m/s$^2$. These values can decrease to 0.3 to 100 m/s$^2$ if geometrical irregularities are taken into account.

Lacave et al. [2000] carried out in situ measurements to determine the range of fundamental natural frequencies and structural damping of typical speleothems. It turned out that only exceptionally long and thin speleothems have natural frequencies within the range of seismic excitation, say below 30 Hz (see Fig. 1). This means that most of them do not undergo dynamic amplification phenomena during seismic motion. They move along with their basement as a rigid structure. However, those few having natural frequencies within the seismic range may undergo significant dynamic amplification, of a factor of 4 or 5, due to extremely low structural damping of the order of 0.1% critical damping. It seems that speleothems can

![Fig. 1. Estimation of the natural frequency of a speleothem as a function of its type and length.](image-url)
only be expected to break during an earthquake either if they are exceptionally long and thin or at particularly weak sections due to structural anomalies. Hence, a statistical approach is mandatory. This means that many more material tests are necessary other than those that were carried out so far in order to constrain the distribution of the bending tensile rupture stress in speleothems.

The general objective of the present study is to find out what can be concluded from the observation of broken and unbroken speleothems in quantitative terms. Is it plausible that earthquakes break speleothems? If broken speleothems do indicate past earthquakes — “speleo-earthquakes” —, then is it possible to quantify the “strength” of such earthquakes in some way? And if yes, what would be the uncertainty of this quantification? Can unbroken speleothems define an upper limit of the “strength” for earthquakes that could have ever occurred during the speleothems’ life time?

The first step was to develop vulnerability curves for typical speleothems. To achieve this aim, the bending resistance of a relatively large number of speleothems taken from the pilot cave was determined in the laboratory. This resulted in a probability density function for the rupture stress in bending. In the pilot cave, the geometry of many speleothems was measured in situ by photographic means. A simplified numerical model for the dynamic behaviour of the measured stalactites and soda-straws, allowing for geometrical irregularities and uncertainties, was then developed. Finally, a comparison was made between the stress induced by a seismic event and the rupture stress, giving us a vulnerability curve for each investigated speleothem. It was assumed that the ground motion in a shallow cave is similar to that at the free surface. Hence, the diffraction of the waves by the cave was neglected. This approximation is justified in view of the large uncertainties linked to other aspects of the modelling. In fact, what counts for the present study are the orders of magnitude.

The second step was to estimate whether past earthquakes can be identified and quantified for the pilot cave under study. To achieve this aim, an original statistical approach for incomplete inhomogeneous data was developed.

The cave “Milandre” near Porrentruy in the Swiss Jura, close to the French border, was chosen as the pilot cave. This cave is characterised by a wide variety of speleothems, particularly many stalactites and soda-straws, some of them broken. The history of the exploration of Milandre indicates that none of the investigated galleries was visited by human beings before 1964, because of the existence of some siphons along the river, and the great distances from the natural entrance (before the construction of an artificial pit). It is also impossible, for the same reasons, that animals have ever been to these parts of the cave. This makes it clear, at least, that the broken speleothems observed in these galleries are not broken by human beings or animals. In fact, most of them are re-calcified on the floor, which indicates that they have not been broken recently.
2. Laboratory Resistance Tests on Stalactites

In order to preserve the unique cave environment of Milandre, no intact speleothems were broken. Only pieces lying on the ground that were not re-calcified on the floor were collected within the upper part of the cave. In total, about 30 pieces of broken stalactites were collected.

A fundamental problem could not be circumvented: all the test specimens have fallen down to the ground. Their internal structure could have been damaged, so the measured bending stress might be too low. However, the opposite could be true as well: the stalactites had to break before falling down, and then may have broken up into several pieces at the impact on the floor. It is reasonable to assume that the weakest sections broke at the time. Thus, in today’s bending tests, one can no longer measure the lowest bending resistance. Since calcite is nearly an ideally brittle material, there is little or no damage expected until a sudden fracture develops. This seems to give more weight to the second hypothesis, i.e. that the bending resistance measured on broken stalactites might be — statistically speaking — too high.

In order to avoid any structural disturbance in the test specimens, the bending tests were carried out on stalactites without any previous treatment. The “base” was glued into a metallic ring that could be held by the testing machine (clamped end). The load was applied statically to the thin end (Fig. 2). These tests were carried out at the Swiss Federal Laboratories for Materials Testing and Research. The rupture moment was calculated with respect to the true rupture section which was in many cases not the section at the clamped end. The maximum rupture stress was evaluated with respect to an ellipse matching the rupture section.

The laboratory tests were made on 20 stalactite specimens. This gave 20 bending tensile rupture stresses, henceforth called “rupture stress”, for the calcite material.

Fig. 2. Procedure used to perform the laboratory tests on stalactites.
Table 1. Rupture stress obtained from laboratory tests on 20 stalactite specimens.

<table>
<thead>
<tr>
<th>No</th>
<th>V1</th>
<th>V2</th>
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<td>6.1</td>
</tr>
</tbody>
</table>

Fig. 3. Rupture stress probability density obtained with a kernel estimation of density from the results of the laboratory tests.

(Table 1). Based on these 20 values, a distribution of the rupture stress was estimated. A parameter free kernel estimation of densities [Silverman, 1986] that does not need any a priori for a particular distribution shape was used. The kernel distribution obtained from the laboratory test data is shown in Fig. 3.

The material tests conducted by Cadorin et al. [2001] gave lower values than the ones performed for the present study. Their tests were performed on specimens from only 4 distinct speleothems, whereas here, 20 distinct stalactites were used. This simple comparison between the different rupture stresses, shows the great variability of this parameter (even within the same cave population: see the variability of the results shown in Table 1). It is therefore not possible to consider a single value for the rupture stress in the modelling of the speleothems’ behaviour. This is the reason why a kernel distribution of this parameter was used here (Fig. 3), allowing the variability to be accounted for between different speleothems, as well as along a single speleothem (see Sec. 4.2).

3. Measurements of the Stalactites

An inventory was made in the Milandre cave. The goal was to gain an overview on the speleothem population and to be able to measure later on — as precisely as possible — the geometry of the speleothems. To this aim, digital photos of
most of the important stalactites and long soda-straws were taken, together with a scale. Some four hundred photos were taken during this field campaign. It was then possible to measure their geometry afterwards with sufficient precision on the computer screen. Ten stalactites were measured as precisely as possible (Table 2).

The length and the development of the diameter along the stalactite’s axis were measured. All sections were assumed to be circles. Each stalactite is finally modelled as a succession of \( n \) truncated cones of equal height \( q = L/n \), \( L \) being the total length and \( n \) is chosen to be 8 throughout the whole study. The cone elements are numbered from \( i = 1 \) to \( j = n \) (\( i \)th element is between section \( d_{i-1} \) and section \( d_i \)). Figure 4 shows a schematic shape of a typical stalactite as it is measured.

Obviously, the measurements are characterised by a certain amount of uncertainty. Typically, a standard deviation of 1 mm, supposing a normal distribution, was estimated for the measurements of the diameters.

### 4. Modelling of the Mechanical Behaviour of Stalactites

In a previous study [Cadorin et al., 2001; Delaby and Quinif, 2000], the acceleration necessary to break the stalagmites was estimated. They were modelled as rigid homogeneous cylinders with irregular diameters. In the present study, only
stalactites are considered. These are modelled as a succession of truncated cones. First a model for rigid stalactites with homogeneous resistance was developed. Then the assumption of homogeneous resistance was dropped. Finally, it turned out that the stalactite deformation, with the possibility of dynamic amplification, had to be taken into account.

4.1. Maximum bending stress in a section

If the rupture stress were homogeneous within a given stalactite, and as far as the geometrical shape is not too irregular, seismic breaking should occur at the stalactite’s base. Therefore, the maximum bending stress is evaluated for the base section \( i = 0 \). In the case of a rigid stalactite, the internal bending moment at section \( i = 0 \) due to seismic inertia forces, for a unity acceleration, can be written as:

\[
M_0 = \rho \cdot \sum_{i=1}^{n} [(i - 1) \cdot q + X_{ci}] \cdot V_i,
\]

(1)

where \( \rho \) is the density, which is chosen to be constant and equal to 2600 kg/m\(^3\) for calcite [Kourimsky and Tvrz, 1983], \( V_i \) is the volume of each truncated cone element and \( X_{ci} \) is the distance between section \( i - 1 \) and the centre of gravity. Then, the maximum bending stress is given by:

\[
\sigma_0 = \frac{32}{\pi} \cdot \frac{M_0}{d_0^3}.
\]

(2)

Since stalactites do not necessarily break at their base, the maximum bending stresses must also be calculated for any other section. This can be done by generalising the derivation of the bending stress in the section \( i = 0 \). If the speleothem can break at any section \( br \) located within the \( k \)th cone element (between sections \( d_{k-1} \) and \( d_k \)), as shown in Fig. 4, the internal bending moment at this section, due to seismic inertia forces of unity acceleration, is given by:

\[
\begin{align*}
M_{br} &= \rho \cdot \left( X_{cbr} \cdot V_{br} \right) + \sum_{i=k+1}^{n} [qsl + (i - 1 - k) \cdot q + X_{ci}] \cdot V_i,
\end{align*}
\]

(3)

where \( X_{cbr} \) and \( V_{br} \) designate the distance to the centre of gravity and the volume, respectively, of the incomplete truncated cone element below the breaking section (between section \( br \) and section \( k \)). \( qsl \) is the height of this incomplete element, as shown in Fig. 4. Then, the stress at the section \( br \) can be written as:

\[
\sigma_{br} = \frac{32}{\pi} \cdot \frac{M_{br}}{d_{br}^3},
\]

(4)

where \( d_{br} \) is the diameter of the breaking section.

A simple check was made to determine whether the stress due to the stalactite’s own weight was significant, in comparison with the stress due to a seismic acceleration of 1 m/s\(^2\). This simple calculation showed that the stress due to the stalactite’s
Fig. 4. Left: description of the modelled geometry of stalactites; right: photo of a stalactite with the measuring scale (each square is 20 mm × 20 mm).

own weight is not significant compared to the value of the dynamic stress. Its own weight was therefore neglected throughout the study.

4.2. Resistance heterogeneity

Homogeneous cylinders would break at their base. However, both in the caves and in the material tests, it was observed that only few stalactites break at their base. This implies that the possibility of a stalactite breaking at any section of possible weakness has to be accounted for. It is therefore necessary to model the material resistance heterogeneity along the stalactites.

For the present study, it was assumed that the rupture stress within two different stalactites cross sections are as uncorrelated as they would be for different stalactites of the same cave, provided that the cross sections are sufficiently distant from each other. This assumption seems reasonable since stalactites are composed of many calcite crystals that grow in a heterogeneous way over many centuries (see
Perette, 1999, for detailed considerations about the growth rate of stalagnites and its links with surface environmental changes; and Hill and Forti, 1997, for ranges of the variation of speleothem growth rates).

Unfortunately, it is not known how much does “sufficiently distant” represent. On the one hand, the local size of the crystals, usually of the order of 1 or 2 mm, might represent a minimum. On the other hand, a distance of the order of the diameter, thus a few centimetres, might be an upper bound. In view of the large uncertainty in the knowledge of the true correlation length, a simplified modelling approach was adopted: it is supposed that there is a distance \( l^* \), named “correlation length”, so that for \( l > l^* \), the rupture stress is fully uncorrelated. This parameter reflects the degree of heterogeneity along the stalactite. It is further assumed that the rupture stress is piecewise constant within the elements of length \( l^* \) of a given stalactite. Obviously, the smaller \( l^* \) is, the more independent the values of rupture stress can be found within a given stalactite. And since the rupture stress is assumed to be a random variable with a probability distribution as given in Fig. 3, the smaller \( l^* \) is, the higher the probability that weak sections exist, and consequently, the higher the vulnerability of a given stalactite. A sensitivity analysis was performed on a test stalactite to show the crucial importance of the parameter \( l^* \).

It seems to be difficult, if not impossible, to directly measure the correlation length \( l^* \). An attempt to constrain \( l^* \) to some — admittedly modest — extent is to model the static material tests that were carried out on 20 stalactites. The specimens only broke in 4 cases at less than 5 mm from the clamped base. This can only be explained by a lack of homogeneity in rupture stress due to internal structural irregularities. These static tests could now be modelled by means of a Monte Carlo simulation for different values of \( l^* \), in a similar way as is done for the seismic case (see below). The value of \( l^* \) — or range of values — that simulates the distribution of rupture locations in the static tests would then be an approximation to the “true” correlation length \( l^* \). The basic idea of the Monte Carlo simulation of the static tests is as follows: the static tests are modelled for different \( l^* \). If ruptures appear — in the average — for forces significantly lower than what was attained in the laboratory tests, the model vulnerability is too high, thus \( l^* \) is too small. If, on the contrary, the rupture forces are higher than the results in the laboratory, the model vulnerability is too small, thus \( l^* \) is too big. The most appropriate correlation length turned out to be \( l^* = 33 \) mm, which will be used in the rest of the study.

4.3. Modelling of breaking: vulnerability curves for rigid stalactites

The present chapter is devoted to the calculation of “vulnerability” curves of stalactites. Here, a vulnerability curve is understood as being the probability of breaking as a function of peak ground acceleration (PGA). Such curves are sometimes also called “fragility” curves. PGA was chosen as the parameter describing the “strength” of the earthquake since most stalactites — the ones with fundamental frequencies above the seismic excitation range — undergo stresses directly proportional to PGA.
Vulnerability curves were calculated with the aid of Monte Carlo simulations. All variations of variables (in statistical language: all realisations of random variables) were done with the aid of “stratified” sampling, called “Latin Hypercube Sampling” (LHS). Using the LHS technique, considering \( N \) different values for \( Q \) input parameters only \( N \) computations are needed for providing a reliable estimate of the actual uncertainty in the final results. This method was mainly developed in the field of nuclear safety, where individual runs to simulate the accidental system behaviour needed exorbitant computer time; it was therefore crucial to reduce the number of computations as much as possible. The reader is referred to McKay et al. [1979] and McKay [1988] for a detailed discussion of the merits and limitations of LHS in Monte Carlo simulations.

The following paragraphs explain how the probability of a stalactite breakage is calculated for a unity value of PGA. By repeating this procedure for different PGA’s, the whole vulnerability curve can be easily obtained. \( N = 1000 \) Monte Carlo simulations were performed per investigated stalactite and PGA value. The goal of each simulation is to test whether the stalactite would break. Summing up all simulations that end with breakages, divided by \( N \), gives the probability of breaking, which is a number between 0 and 1. Considering the \( I \)th Monte Carlo simulation (Fig. 5):

- First, the geometry of the stalactite under investigation is defined with the aid of the measured values \( L \) and \( d_i \), varied by random deviations according to the estimated measurement uncertainties (i.e. the stalactite’s geometry is slightly different in each simulation).
- Second, the maximum bending stress is calculated at section 0 (the section with the a priori highest probability of breaking). This stress is compared with a rupture stress value taken at random, but according to the probability density function given in Fig. 3. If the maximum (tensile) bending stress is greater than the rupture stress, the stalactite is considered to break, and the \( I \)th simulation is terminated. If not, a further step is undertaken.
- Third, the next section with an uncorrelated rupture stress is found at a distance \( l^* \) below section 0: section \( br_1 \). The maximum bending stress is calculated at that section; this stress is slightly smaller than the stress at section 0, since the lever of the inertia forces diminishes. A new value for the rupture stress is taken at random from the corresponding probability distribution. Again, if the maximum bending stress is greater than the rupture stress, the stalactite is considered to break, and the \( I \)th simulation is terminated. If not, a further section, at a distance \( l^* \) further down, is tested, and so on.

The diameter of section \( br_1 \), lying between sections \( k-1 \) and \( k \), is first calculated for an ideal truncated cone element, based on the (randomly varied) diameters \( d_{k-1} \) and \( d_k \). Then, an additional variation is introduced at section \( br_1 \), in order to account for a diameter irregularity deviating from an ideal truncated cone.
Fig. 5. Procedure used to model the mechanical behaviour of stalactites, taking into account the heterogeneity of the material and uncertainty on the measured geometrical features.

If it turns out that no section breaks from section 0 down to the free end, the stalactite is regarded as unbroken.

4.4. **Elastic stalactites: dynamic amplification**

*In situ* measurements of the natural frequencies and damping of speleothems [Lacave et al., 2000] show that the fundamental natural frequencies of most speleothems that can be found in caves are well above the range of seismic excitation, usually supposed to be limited to $f < \sim 30$ Hz. The consequence is that there would be no dynamic amplification of the movement of the speleothem with respect to its base, i.e. the speleothem would follow the earthquake ground motion as a rigid body. This is the reason why the stalactites were first modelled as rigid bodies in this study. However, since the long and slender speleothems are those with the highest probability of breaking, the vulnerability of a large part of those speleothems that break would be significantly underestimated if dynamic amplification were not taken into account. The conclusion is that although most of the speleothems react like rigid bodies, it is mandatory to model dynamically those that do undergo dynamic amplification.

Modest local earthquakes can show their maximum dynamic amplification in the response spectrum for frequencies as high as $\sim 15$ Hz, $\sim 20$ Hz or even $\sim 25$ Hz. Large remote earthquakes may show something like a “plateau” of maximum amplification.
between \(\sim 2\) Hz and \(\sim 10\) Hz, similar to usual design spectra for rock sites. For the present study, a very simplified approach was adopted. It was assumed that any stalactite with estimated natural frequency \(f_0 < 25\) Hz would undergo full dynamic amplification according to the “plateau” value of a typical response spectrum. Since speleothems with \(f_0 < \sim 2\) Hz are extremely rare and do not exist in the pilot cave, it was not necessary to account for the lower amplifications that can be found in the low frequency range of typical response spectra. Supposing full dynamic amplification within the whole frequency range of \(2\) Hz \(< f_0 < 25\) Hz obviously leads, statistically speaking, to an overestimation of the probability of breaking. In fact, only part of the speleothems with natural frequencies within this frequency range would actually undergo full dynamic amplification during a real earthquake. However, it is suggested to refine this aspect in a later study.

A simple approach was used in order to choose a constant amplification factor. The Eurocode 8 elastic response spectra are characterised by a plateau amplification of \(2.5 \cdot \eta\), with a damping correction coefficient \(\eta\) given by:

\[\eta = \sqrt{\frac{\xi}{2 + \xi}},\]  

where \(\xi\) is the percentage of critical damping [Eurocode 8, 1994]. From in situ measurements of the damping of speleothems [Lacave et al., 2000], a damping value of about 0.1% of the critical value can be inferred for stalactites. For \(\xi = 0.1\%\), it follows that \(\eta = 1.8\). Hence, an amplification factor of \(2.5 \cdot \eta = 4.5\) is used for each speleothem characterised by a natural frequency below 25 Hz.

In the case of dynamic amplification, the bending moment is calculated by means of “equivalent static forces” that correspond to the inertia forces due to the vibration in the fundamental mode (see e.g. Chopra, 2001). Only the fundamental mode was taken into account for the following reason: the frequency of the 2nd mode of vibration is 6.4 times the fundamental frequency which is the fundamental frequency for a prismatic cantilever beam. Hence, if the fundamental mode undergoes full dynamic amplification, it is highly improbable that the 2nd mode of vibration undergoes significant dynamic amplification as well. Thus, the contribution of the higher modes remains comparatively small.

A test made on stalactite stt02 clearly showed that for speleothems that have natural frequencies in the frequency range of seismic excitation, not taking into account dynamic amplification would lead to a drastic underestimation of their vulnerability (Fig. 6).

4.5. Resulting vulnerability curves

A set of ten stalactites of various types and shapes were chosen as sample stalactites. It is to be noted that stalactites stt02, stt04, stt06 and stt07 are affected by dynamic amplification phenomena due to their relatively low natural frequencies. The vulnerability curves for each of these stalactites were calculated, as shown on Fig. 7. These results reveal the fact that earthquakes can break stalactites. For
Fig. 6. Vulnerability curves for the test stalactite, stt02, under the hypothesis of a rigid body (dashed line) or accounting for full dynamic amplification of the motion (solid line).

Fig. 7. Vulnerability curves of the ten test stalactites.
example, the most vulnerable stalactites are broken with an 85% probability at an acceleration of 2.5 m/s$^2$ and 40% of some less vulnerable ones are broken for the same acceleration.

Long and slender speleothems are the most vulnerable ones for three concurrent reasons:

1. purely geometrical influence: the longer the speleothem, the greater is the lever of the inertia forces and herewith the bending moment. The smaller the diameter, the higher is the maximum tensile bending stress for a given bending moment,
2. dynamic amplification, since sufficiently long and slender speleothems have their fundamental natural frequencies within the frequency range of seismic excitation,
3. the longer the speleothems, the higher the probability that they contain a weak section, i.e. an internal structural irregularity with a low rupture stress.

In view of the large uncertainties associated with the modelling of the speleothems’ mechanical behaviour, it would not make sense to measure hundreds of speleothems in detail and to calculate their vulnerability curves individually. Therefore, it was decided to define four classes of vulnerability for the statistical analysis of the pilot cave:

- Class 1: very vulnerable
- Class 2: vulnerable
- Class 3: little vulnerable
- Class 4: not vulnerable.

Figure 8 gives the vulnerability curves associated with these classes. If this classification is felt to be too coarse, it can be theoretically refined. However, it would then become difficult to associate individual speleothems to the different classes.

![Vulnerability curves](image)

Fig. 8. Vulnerability curves for the four vulnerability classes of stalactites.
without again measuring them in detail and calculating their vulnerability curves individually.

5. Statistical Analysis of the Stalactite Population

The final objective is to find out

- whether past earthquakes can be identified and quantified for the pilot cave under study, and
- whether an upper limit of acceleration can be estimated which has never been exceeded during the stalactite’s lifetime with a certain probability.

To answer these questions, an original statistical approach for incomplete inhomogeneous data was developed. Since the theory of this approach is far from being elementary, its development is not given in detail here. Only the underlying hypotheses, the results and their interpretation are given in the main body of the present paper. So far, the statistical analysis has been done only in a time independent way. This means that the stalactite’s growth over time has not been explicitly considered. This important aspect would have to be treated in a later study. In this study, the observed stalactites, with their present shape, are simply assumed to have a certain age, henceforth referred to as “observation time”. Furthermore, it is assumed that seismic events that produce a PGA in the cave of less than \( \alpha_0 = 0.5 \text{ m/s}^2 \) cannot be identified, since they would hardly break any speleothem. Therefore, “seismic events”, by definition, are only events that exceed this threshold.

5.1. Prior information

The statistical approach is mainly based on Bayesian statistics. That is why it asks for an a priori knowledge of the result, called “prior”. If nothing is known a priori, the prior can be kept non-informative, and the final result is only influenced by the observed data. In the present case, the subjective evaluation of the cave is that “probably at least one event has occurred in the last 1000 years over the absolute threshold of PGA”; this absolute threshold was set to 0.5 m/s\(^2\). This impression is based on the fact that the cave is situated in a moderately seismic region and that fault mirrors exist in the cave. Therefore, a probability of 70\% for one event or more, and 30\% for no seismic event during the last 1000 years was chosen as a prior for the main statistical analysis (run 1 of the statistics programme). Sample data confirmed this impression since otherwise, the 54 stalactites observed as broken would have had to be broken by non-seismic reasons.

In order to test the importance of this prior in the present context, an additional statistical analysis was done with a prior assuming a probability of 100\% that no seismic event has occurred in the last 1000 years (run 2 of the statistics programme).
Figure 9 shows the prior obtained from these prior subjective evaluations. The prior probability density for both priors (run 1 and run 2) are shown. They should represent our prior knowledge on the occurrence rate of events over the absolute threshold $0.5 \text{ m/s}^2$. This prior information is completed with additional important specifications:

- A surely attainable level of PGA, i.e. this level of PGA must be a possible one for any assumed distribution of PGA. This was assessed as $2 \text{ m/s}^2$.
- An almost impossible level of PGA, i.e. the PGA which, being higher than $0.5 \text{ m/s}^2$, cannot be attained with more probability than $10^{-7}$. This was assessed as $15 \text{ m/s}^2$.
- There is some unknown physical upper limit of possible PGA.

These prior parameters were used to determine the acceptable range of parameters of the probability distribution of PGA. This distribution was assumed to fit approximately a generalised Pareto distribution. A similar prior assessment of such parameters was presented in Egozcue and Ramis [2001].

5.2. Observation of the stalactite population

The stalactites were sampled in the most exhaustive way possible, by photographic means. This sampling was limited to the parts of the cave with the most dense stalactite population. The sampled stalactite population was divided into the four vulnerability classes defined in Fig. 8. Furthermore, two groups of stalactites are considered: the unbroken and the broken ones.
Table 3. Classification of the unbroken stalactite population.

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<th>class 3</th>
<th>class 4</th>
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<td>90%</td>
<td>10%</td>
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<td>10%</td>
<td>90%</td>
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<td>188</td>
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It is not obvious to determine which vulnerability class a stalactite belongs by only looking at its geometry. This is the reason why, for each class of stalactites, a probability is given that an individual stalactite associated to this class would actually belong to another class (the total probability of the 4 classes being 100%). Table 3 shows this “fuzzy” classification of the unbroken stalactites (total number 324).

The broken stalactites (total number 54) were also investigated, and for each of them, a probability of belonging to class 1 to 4 was given. Furthermore, it is taken into account that some of the apparently unbroken stalactites might in fact have been broken, but no visible traces are present today (the broken section is fully hidden by new calcification). In an analogous way, stalactites observed as seismically broken can be broken by something else other than an earthquake or can they actually be unbroken (sudden change in growth rate or type). These uncertainties are taken into account as follows:

- all stalactites observed as unbroken are assumed to be with 95% probability really unbroken and at 5% broken;
- all stalactites observed as broken are assumed to be:
  (a) for Class 1 and 2: 65% broken by an earthquake and 35% broken by something else or unbroken;
  (b) for Class 3: 60% broken by an earthquake and 40% broken by something else or unbroken;
  (c) for Class 4: 50% broken by an earthquake and 50% broken by something else or unbroken.

It has to be pointed out that it is difficult to discriminate between several possible speleothem breaking causes [Gilli, 1999; Delaby and Quinif, 2000]. With careful local field observations, it is generally possible to rule out some of these causes, but one can rarely exclude all possible non-seismic causes. It is to account for this uncertainty that a probability of breaking by a non-seismic cause has been estimated, for all observed broken stalactites.
5.3. Results: Probably attained PGA in the cave

The main result of the statistical analysis is the estimation of the probability of occurrence of 0 events over a given threshold PGA — i.e. the PGA threshold was not attained — in an also given period of time; in this case the hypothetical 5000 years.

Figure 10 shows such results for run 1 and run 2. Both prior and posterior are shown. A comparison of both prior results point out that, in fact, prior information was different. For low thresholds (0.5 up to 3.5 m/s^2) run 1 prior predicted less probability of 0 events than run 2 prior. The role is inverted for higher thresholds and finally both converge to 1.0 near the considered upper limit of 20 m/s^2. After Bayesian updating, these probabilities maintain the mentioned features and shapes but the probabilities of 0 events have been increased, slightly more in run 1 than in run 2. Differences between the two posterior probabilities (run 1 and 2) are due to the differences between the two priors. The first prior (run 1) may be intuitively interpreted as “it is vaguely likely that, in a 1000-year observation time, we would observe at least one event over 0.5 m/s^2 and preferably the events, if any, were not very large”. The second prior (run 2) conveys information approximately equivalent to: “we are sure that there were no events over 0.5 m/s^2 in a 1000-year observation time”. When comparing these two priors it appears that, for low values of PGA, say under 3.5 m/s^2, the first prior gives a lower probability of non-exceedance of 0.5 m/s^2 as compared to the second prior. However, the second prior does not distinguish between the different PGA: all of them were similarly unlikely.

After these two different priors, the data confirmed to a certain extent that one not very large event might have occurred. The Bayes updating interpreted the
posterior curves of non-exceedance probabilities (Fig. 10) to grow less in the low values of the PGA than in the higher ones. This is more apparent in run 1, where the information coming from the data is in better agreement with the prior than it is in the second run.

Figure 11 shows a secondary but intuitive result. The probability density of the upper limit of the acceleration (PGA) is shown for both runs and for prior and posterior. Run 1 prior shows a preference of an upper limit of the PGA between, say, 5 and 8 m/s², whereas run 2 prior shows no preferences between the absolute threshold (0.5 m/s²) and the maximum admissible value (20 m/s²). After the Bayesian updating, both posterior densities show that the more likely values of the upper limit of the PGA are less than, say, 6 m/s². This clearly points out:

• the data were informative as both priors were changed in the same direction;
• probably, no really strong event occurred and, also probably, a moderate event with PGA between 0.5 and 2.5 m/s² occurred, with the upper limit of the PGA most likely at 2.5 m/s² (posterior 2) or at about 3 to 4 m/s² (posterior 1).

5.4. Discussion of the results

It is interesting to compare the results obtained with the history of regional seismicity that is known — as far as strong events are concerned — for about 700 years. In fact, the only strong event known for that period is the 1356 Basel earthquake. Its epicentre was about 40 km away from the Milandre cave. In the context of a recent re-evaluation of the Swiss historical seismic catalogue, a magnitude value of $M_w = 6.9$ was estimated for the Basel earthquake, by the Swiss Seismological Service [Fäh et al., 2002]. According to Ambraseys et al. [1996], a mean value for
the larger component of the horizontal PGA of 1 m/s$^2$ would be expected in this case on rock outcrop. Since the Milandre cave is situated about 30 m below ground level, this value would also be approximately valid in the cave.

Since the standard deviation of the PGA attenuation law is at a factor of 1.8, the actual occurred PGA value may have been also significantly higher. This is even probable. It is known from Mayer-Rosa and Cadiot [1979] as well as from Levret et al. [1996] that the isoseismal counters were elongated in the direction from the epicentre to the region of Milandre, i.e. the attenuation in this direction was significantly less than the mean. Furthermore, both publications show Milandre within the $I_{MSK} = VIII$ isoseismal counter. This results in a PGA-value of 1.8 m/s$^2$, corresponding to the mean plus one standard deviation, much more probable than a value of 1 m/s$^2$.

6. Remarks Concerning the Use of Soda-Straws in a Statistical Approach

Concerning the classification of the soda-straw population, in view of their use in the statistical approach, two main problems appear:

- the apparently unbroken ones might have been broken in the past, with an unknown probability. This is not visible because of the tube shape of a soda-straw: a broken soda-straw can continue to grow without leaving any visible marks.
- it is not possible to determine to which class the broken ones belong.

Therefore:

- Contrary to what has been suggested by some authors, it seems not possible to conduct quantitative paleo-seismic investigations with the use of soda-straws (broken or not). The only information that can be obtained from observed broken soda-straws is the information given by the prior: “it is probable that there was an event in this area . . .” no calculation of the probable attained acceleration seems possible.
- Soda-straws can, however, be used for case studies on recent known earthquakes, for which the soda straw population is known before and after the earthquake. In such cases, it would be interesting to estimate the maximum (and minimum) attained acceleration in an area, in order to test the theory presented in this paper.

7. Case Study Basel Earthquake

7.1. Cave of Bättlerloch

The cave called Bättlerloch is located near Zwingen (Switzerland) at about 5 km south of the supposed fault of the 1356 Basel earthquake [Meghraoui et al., 2001].
Fig. 12. Two views of the same group of broken stalactites in Bättlerloch cave.

The cave is sub-horizontal and has a total length of 1000 m. This cave is characterised by a very low amount of speleothems. Nevertheless, it contains a group of broken stalactites, which is located on the side of the active main gallery. This group is composed of a dozen of stalactites, all broken, with a soda-straw re-growth, as shown in Fig. 12. A case study was carried out to determine the acceleration necessary to break this group of speleothems.

7.2. Acceleration necessary to break the stalactite group of Bättlerloch

The case study conducted on the group of broken stalactites in Bättlerloch is based on the following hypotheses:

- the stalactites have been broken by an earthquake (as the gallery is semi-active, they could also have been broken by a sudden flood in the past, at a time when the river activity was stronger);
- before the earthquake, their length was the maximum possible length down to the present riverbank.

The group is composed of 12 specimens with a maximum possible original length of about 60 cm and a diameter between 1 and 2 cm. This shape is equivalent to that of the sample stalactite st109. The corresponding natural frequency is about 30 Hz. In the main study, this stalactite was considered as having a static behaviour,
because dynamic amplification was only considered for stalactites with natural frequencies under 25 Hz. Since the source of the 1356 Basel earthquake was very close to the cave (supposed fault at about 5 km), one can also expect some dynamic amplification of the motion up to higher frequencies, such as 30 Hz. Two calculations were made: in the static and in the dynamic cases (considering amplification by a factor 4.5). These results indicate that 100% of the stalactites of this type are broken with an acceleration of 10 m/s$^2$, if dynamic amplification is considered, due to the proximity of the epicentre.

### 7.3. Comparison with the possibly attained acceleration during the Basel earthquake

Most of the broken stalactites in the group are characterised by a soda-straw regrowth. From one stalactite to the other, these soda-straw pieces vary from 2 to 24 cm, with a mean value of about 10 cm.

Considering that the growth rate for soda-straws in the cave of Choranche (Vercors, France) is about 1 to 2 cm per century, it would be plausible — from the time point of view — that these soda-straws started growing after the 1356 Basel earthquake, i.e. about 650 years ago.

The cave of Bättlerloch is located 5 km from the supposed fault. The intensity experienced in the area is IX–X. Considering the Basel earthquake, a mean value for the larger component of the horizontal PGA of 5.7 m/s$^2$ would be expected in this case on rock outcrop (Ambraseys et al. [1996]). Since the Bättlerloch cave is subhorizontal and is situated only at a few tens of metres below the ground level, this value would also be approximately valid in the cave. Since the standard deviation of the PGA attenuation law is a factor of 1.8, the actual occurred PGA value may have been also significantly higher. The mean plus one standard deviation value of the PGA would be 10.3 m/s$^2$. Such a PGA could explain the breaking of 100% of the observed broken stalactite group.

### 7.4. Remarks on the sensitivity with respect to the original stalactites’ length

As no broken pieces are available on the floor, the whole case study above was made under the hypothesis that the original length of the broken stalactites was the maximum possible length (60 cm), down to the river bank, as it is nowadays. If, on the contrary, a length of only 30 cm is estimated for these stalactites, they are comparable to the test stalactite stt10 with a fundamental resonance frequency of about 50 Hz, so most probably there is no dynamic amplification. Only about 6% of such stalactites can be broken by an acceleration of 10 m/s$^2$. This would lead to the conclusion that the 1356 Basel earthquake would not have broken 100% of the stalactite group observed in the Bättlerloch cave.

The conclusions are:
• This simple example shows that it is impossible to make any reliable quantitative estimation of the acceleration in the case of observed broken speleothems when nothing makes it possible to determine their original length (fallen pieces on the floor, possibly similar unbroken specimens nearby, etc.).
• It is possible that the 12 stalactites were broken by the 1356 Basel earthquake, but only if two concurrent hypotheses that increase the rupture probability are assumed: the maximum possible length of all 12 stalactites (improbable) and a particularly high PGA.

8. General Conclusions

It is reminded here that, in the framework of this rather qualitative study, the accelerations in shallow caves were considered to be approximately the same as that at the free surface. The diffraction of the waves produced by the cave can be neglected in view of the uncertainties that had to be dealt with for the modelling of the speleothems’ behaviour under seismic action. The main tentative conclusions that can be drawn so far are:

• Most of the existing stalactites (and stalagmites), except long and slender ones, hardly break during realistic earthquakes, say with PGA < 10 m/s^2.
• Nevertheless, there are many broken stalactites present in the pilot cave where low or very low seismic vulnerability is inferred for their estimated unbroken shape. It seems therefore improbable that the majority of them broke during a seismic event.
• Most of the long and slender speleothems are expected to break during a “reasonably” strong earthquake, say with 3 m/s^2 < PGA < 10 m/s^2.
• The observed data nevertheless indicate that probably at least one seismic event has occurred. (As long as no datation of the broken speleothems is undertaken, multiple events cannot be distinguished from a single event.) The most probably attained PGA was of the order of 1 to 2 m/s^2.

However, this statement must be understood as a rather “weak” information. The 1356 Basel event is thought to have produced a PGA on rock between 1 and 2 m/s^2 in the pilot cave.

The co-existence of intact, but vulnerable speleothems with broken ones of very low vulnerability is not necessarily a contradiction. As long as no datation has been undertaken, it cannot be ruled out that the unbroken vulnerable speleothems have appeared later than the last strong earthquake that had broken the less vulnerable ones.

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References


